Note: In this problem set, expressions in green cells match corresponding expressions in the text answers.

```
Clear["Global`*"]
```

1. Let f[x, y] = k when 8 ≤ x ≤ 12 and 0 ≤ y ≤ 2 and zero elsewhere. Find k. Find P(X ≤ 11, 1 ≤ Y ≤1.5) and P(9 ≤ X ≤ 13, Y ≤ 1).

I can define a rectangle as a region. This will be a continuous function.

```
d1 = Rectangle[{8, 0}, {12, 2}];
```

And test it.

```
RegionMember[d1, {x, y}]
```

```
(x | y) ∈ Reals && 8 ≤ x ≤ 12 && 0 ≤ y ≤ 2
```

I can define f[x, y] as a piecewise function

```
f[x_, y_] = Piecewise[{{k, {x, y} ∈ d1}, {0, {x, y} ∉ d1}}]
```

```
{ k  {x, y} ∈ Rectangle[{8, 0}, {12, 2}]
  0  True
```

Checking the area.

```
4 × 2
```

```
8
```

Knowing the area of the rectangle, I can update the function definition

```
f[x_, y_] = Piecewise[{{0.125, {x, y} ∈ d1}, {0, {x, y} ∉ d1}}]
```

```
{ 0.125  {x, y} ∈ Rectangle[{8, 0}, {12, 2}]
  0      True
```

Having the PDF, I should be able to come up with some automation from Mathematica, but I haven't found it yet. For this particular problem, it is a simple operation, and I just do the simple calculations. For X ≤ 11, 1 ≤ Y ≤1.5) I can find the probability of the volume

```
3 × 0.5 × 0.125
```

```
0.1875
```

And for 9 ≤ X ≤ 13, Y ≤ 1 there is peculiarity, because x = 0 when beyond 12. So that side is only 3 units wide.

```
3 × 1 × 0.125
```

```
0.375
```

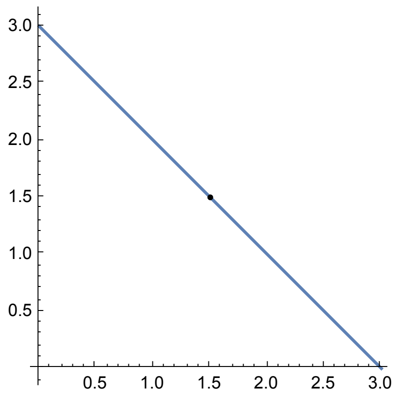
The green cells above match the answer in the text.

3. Let f[x, y] = k if x > 0, y > 0, x + y < 3 and 0 otherwise. Find k. Sketch f[x, y]. Find P(X + Y ≤ 1), P(Y > X).

```
Clear["Global`*"]
```

This is similar to the last problem.

```
Plot[3 - x, {x, 0, 3}, AspectRatio → Automatic,
 ImageSize → 200, Epilog → {Point[{1.5, 1.5}]}]
```



I can make a 2D domain of a graphic region.

```
d2 = Polygon[{{0, 0}, {3, 0}, {0, 3}, {0, 0}}]
Polygon[{{0, 0}, {3, 0}, {0, 3}, {0, 0}}]
```

I can test the region to make sure it is recognized.

```
RegionMember[d2, {x, y}]
(x | y) ∈ Reals && 3 y ≥ 0 && -3 (-3 + x) - 3 y ≥ 0 && 3 x ≥ 0
```

I can define a function using the interior of the region as domain. This is another continuous function with a constant function value.

```
f[x_, y_] = Piecewise[{{k, {x, y} ∈ d2}, {0, {x, y} ∉ d2}}]
{ k  {x, y} ∈ Polygon[{{0, 0}, {3, 0}, {0, 3}, {0, 0}}]
  0  True
```

The area of the region is

```
Area[d2]
```

$$\frac{9}{2}$$

allowing me to update the function with the simple volume.

```
f[x_, y_] = Piecewise[{{2/9, {x, y} ∈ d2}, {0, {x, y} ∉ d2}}]
```

$$\begin{cases} \frac{2}{9} & \{x, y\} \in \text{Polygon}[\{\{0, 0\}, \{3, 0\}, \{0, 3\}, \{0, 0\}\}] \\ 0 & \text{True} \end{cases}$$

As for the first probability, it is a simple volume calculation, as in the last problem: P (X + Y <= 1)

$$\frac{1}{2} * \frac{2}{9}$$

$$\frac{1}{9}$$

The second probabilty focuses on a triangle P (Y > X). The point in the plot above suggests the symmetry which simplifies the volume calculation.

$$1.5 \times 1.5 \times \frac{2}{9}$$

$$0.5$$

The green cells above match the answer in the text.

5. Find the density of the marginal distribution of Y in figure 524.

Referring to example 2 and figure 524 on p. 1053, I see that α_1 and β_1 are on the x-axis, and the other two on the y-axis.

This is a continuous uniform distribution. Numbered line (8) in the example explains that

$$(\beta_1 - \alpha_1) * (\beta_2 - \alpha_2) = k$$

and that

$$f[x, y] = \frac{1}{k}$$

Numbered line (16) on p. 1055 concerns finding the marginal distribution of Y, the topic of this problem, and states that

$$f_2[y] = \int_{-\infty}^{\infty} f[x, y] \, dx$$

The above can be transformed into

$$\int_{\alpha_1}^{\beta_1} \frac{1}{(\beta_1 - \alpha_1) * (\beta_2 - \alpha_2)} \, dx$$

the limits of integration changed from their improper integral values because I can only integrate within the domain of f[x, y]. Since the integrand is a constant, the result of the integration is

$$\frac{(\beta_1 - \alpha_1)}{(\beta_1 - \alpha_1) * (\beta_2 - \alpha_2)} = \frac{1}{(\beta_2 - \alpha_2)}$$

and this result agrees with the answer in the text.

7. What are the mean thickness and the standard deviation of transformer cores each consisting of 50 layers of sheet metal and 49 insulating paper layers if the metal sheets have mean thickness 0.5 mm each with a standard deviation of 0.05 mm and the paper layers have mean 0.05 mm each with a standard deviation of 0.02 mm?

I do not have a plan to solve this. It seems like it will not be possible to get an exact answer. But I have made two groups using the input from the problem description.

```
datam = RandomVariate[NormalDistribution[0.5, 0.05], 50]
```

```
{0.558216, 0.566597, 0.538682, 0.4675, 0.541684, 0.550354, 0.489031,
 0.478492, 0.487061, 0.467497, 0.519695, 0.396914, 0.521427,
 0.511171, 0.444294, 0.455204, 0.499849, 0.504318, 0.516317,
 0.457775, 0.408534, 0.577282, 0.501376, 0.468424, 0.409549,
 0.506527, 0.372988, 0.522404, 0.508176, 0.543568, 0.529384,
 0.499485, 0.512613, 0.452279, 0.530811, 0.529966, 0.503148,
 0.484531, 0.45554, 0.503649, 0.607374, 0.485367, 0.436549, 0.519838,
 0.430133, 0.534794, 0.530962, 0.479134, 0.510001, 0.546354}
```

```
part1 = Total[datam]
```

```
24.8728
```

```
m1 = Mean[datam]
```

```
0.497456
```

```
StandardDeviation[datam]
```

```
0.0474566
```

```
datap = RandomVariate[NormalDistribution[0.05, 0.02], 49]
```

```
{0.0395329, 0.0466633, 0.0180818, 0.0265622, 0.0500569,
 0.0452602, 0.0393113, 0.0664617, 0.0619587, 0.0417175,
 0.0361719, 0.0615357, 0.0458454, 0.0463132, 0.0329824,
 0.0627634, 0.0528396, 0.0712693, 0.0676439, 0.0457183,
 0.050865, 0.0251026, 0.0664769, 0.0719348, 0.0415759, 0.0675027,
 0.0726362, 0.0549021, 0.0472956, 0.0485101, 0.0372773,
 0.075856, 0.0752395, 0.0575842, 0.0612874, 0.0539422, 0.0684779,
 0.0620852, 0.036947, 0.0510526, 0.0452759, 0.0371434, 0.0541386,
 0.0681146, 0.0659026, 0.0525161, 0.0734824, 0.0470252, 0.0331304}
```

```
part2 = Total[datap]
```

```
2.56197
```

```
m2 = Mean[datap]
0.0522851

StandardDeviation[datap]
0.0143689

part1 + part2
27.4348
```

Above in yellow I added the two sets of randomly generated data. As the text answer is 27.45, I think it is not a bad result. The text answer for standard deviation is 0.38 mm. This is so large I don't know what to do with it. The text theorem 3, p. 1058, discusses ways to get standard deviations for multivariate distributions from variances, and if I was really interested in getting the text answer I would pursue this.

$$\text{proposeddeviation} = \frac{50\,(0.04745) + 49\,(0.01436)}{99}$$

```
0.0310721
```

11. A 5-gear assembly is put together with spacers between the gears. The mean thickness of the gears is 5.020 cm with a standard deviation of 0.003 cm. The mean thickness of the spacers is 0.040 cm with a standard deviation of 0.002 cm. Find the mean and standard deviation of the assembled units consisting of 5 randomly selected gears and 4 randomly selected spacers.

This problem should be done in some multivariate form, else the spirit of the section is being ignored. But I go on anyway with

```
Clear["Global`*"]
```

I mix up a batch of gears using the specification.

```
datag = RandomVariate[NormalDistribution[5.020, 0.003], 5]
{5.01884, 5.01166, 5.02166, 5.02156, 5.01753}
```

and add up the total thickness of the virtual gears

```
part1 = Total[datag]
25.0913
```

and calculate the mean (though it won't be used).

```
m1 = Mean[datag]
5.01825
```

The standard deviation has increased slightly from the original, which seems reasonable.

```
sd1 = StandardDeviation[datag]
```

0.00408888

Now I make a set of spacers, using their spec

```
datas = RandomVariate[NormalDistribution[0.040, 0.002], 4]
```

{0.0373853, 0.0412382, 0.0409712, 0.0430944}

and add up the total thickness.

```
part2 = Total[datas]
```

0.162689

And check the mean (again, not to be used).

```
m2 = Mean[datas]
```

0.0406723

Again the standard deviation has increased slightly from the original.

```
sd2 = StandardDeviation[datas]
```

0.00238609

Adding up the assembly thickness, I see it is pretty close to the text answer, which is 25.26.

```
totg = part1 + part2
```

25.2539

The combination of standard deviations follows from something I found on some site, but it is less than half the standard deviation shown in the text answer (0.0078 cm).

$$\frac{(5\ \text{sd1} + 4\ \text{sd2})}{9}$$

0.00333209

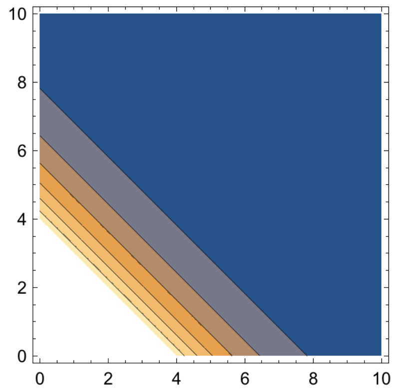
Again I take the opportunity to shine on the text method of calculating bivariate standard deviations.

13. Find P(X > Y) when (X, Y) has the density f[x, y] = 0.25 $e^{-0.5\,(x+y)}$ if x ≥ 0, y ≥ 0 and 0 otherwise.

```
Clear["Global`*"]
```

I can plot the pdf in a contour plot.

```
ContourPlot[0.25 e^(-0.5 (x+y)), {x, 0, 10}, {y, 0, 10}, ImageSize → 200]
```



I can check to see where the pdf is normalized.

```
Table[Integrate[0.25 e^(-0.5 (x+y)), {x, 0, k}, {y, 0, k}], {k, 30, 40}]
{0.999999, 1., 1., 1., 1., 1., 1., 1., 1., 1., 1.}
```

I can express the domain of the bivariate pdf as a region.

```
d2 = Region[x ≥ 0 && y ≥ 0, {x, y}]
Region[x ≥ 0 && y ≥ 0, {x, y}]
```

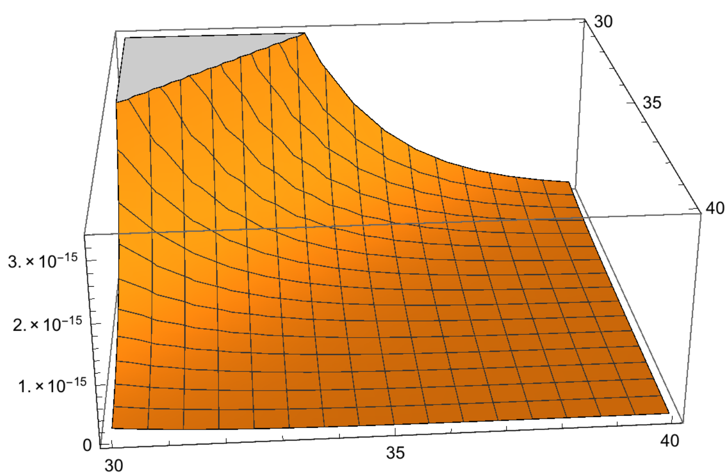
I can define the pdf in terms of the domain region.

```
f[x_, y_] = Piecewise[{{0.25 e^(-0.5 (x+y)), x ≥ 0 && y ≥ 0}, {0, x < 0 && y < 0}}]
```

$$\begin{cases} 0.25\, e^{-0.5\,(x+y)} & x \geq 0 \;\&\&\; y \geq 0 \\ 0 & \text{True} \end{cases}$$

I plot the pdf to see that Mathematica is responding to the function definition.

```
Plot3D[f[x, y], {x, 30, 40}, {y, 30, 40}]
```



I try to roll a custom distribution to incorporate the pdf, but it doesn't seem functional.

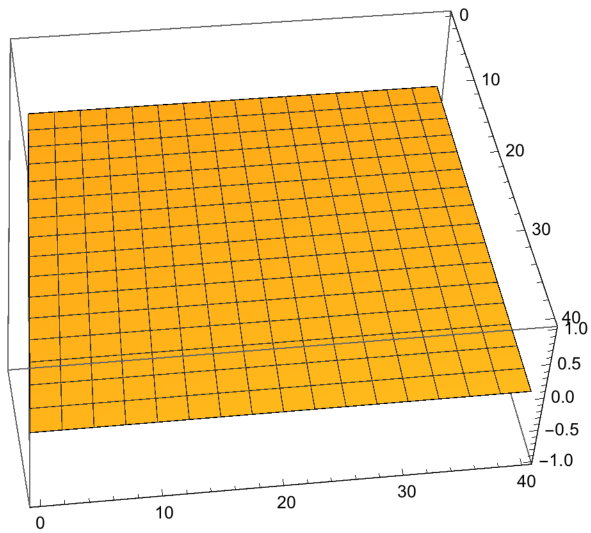
```
d1 = ProbabilityDistribution[f[{x, y}], {x, 0, 1}, {y, 0, 1}]
ProbabilityDistribution[f[{x1, x2}], {x1, 0, 1}, {x2, 0, 1}]
```

I try a different standard distribution, particularized to the area where the pdf has shown consistency.

```
d2 = BinormalDistribution[{35, 35}, {0.001, 0.001}, 0]
BinormalDistribution[{35, 35}, {0.001, 0.001}, 0]
```

As shown in the plot below, this distribution has a pdf with no character. The comparison of particular input values must be left to the mean and stand. dev. within the distribution.

```
Plot3D[Evaluate[PDF[d2, {x, y}]], {x, 0, 40},
 {y, 0, 40}, Exclusions → None, PlotRange → All]
```



I request the diagnostic probability.

```
Probability[x - y > 0, {x, y} ≈ d2]
```

```
0.5
```

I don’t really know if I showed anything or not, but the green cell matches the answer in the text.

```
lisdis =
  Table[BinormalDistribution[{n, n}, {0.001, 0.001}, 0], {n, 35, 99}];
Length[lisdis]
65
```

I wanted to take a longer view but the following input line would not process.

```
Table[Probability[x - y > 0, {x, y} ≈ lisdis[[n]]], {n, 1, 65}]
$Aborted
```

15. Give an example of two different discrete distributions that have the same marginal distributions.

17. Let (X, Y) have the probability function f[0, 0] = f[1, 1] = $\frac{1}{8}$, f[0, 1] = f[1, 0] = $\frac{3}{8}$. Are X and Y independent?

No they are not independent. All the possible occurrences in the distribution are taken up at the four points listed. The points being locked, the coordinates making up the points are likewise locked.